COSTA: Contribution Optimizing Sales Territory Alignment

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Abstract

The alignment of sales territories has a considerable impact on profit and represents a major problem in salesforce management. Practitioners usually apply the balancing approach. This approach balances territories as well as possible with respect to one or more attributes such as potential or workload. Unfortunately, this approach does not necessarily guarantee maximizing profit contribution. Thus, it does not provide an evaluation of the profit implications of an alignment proposal in comparison with the existing one. In consequence, several authors proposed nonlinear integer optimization models in the 1970s. These models attempted to maximize profit directly by considering the problems of allocating selling time (calling plus travel time) across accounts as well as of assigning accounts to territories simultaneously. However, these models turned out to be too complex to be solvable. Therefore, the authors have either approximated the problem or proposed the application of heuristic solution procedures on the basis of the suboptimal principle of equating marginal profit of selling time across territories.

To overcome these limitations, we propose a new approach, COSTA, an acronym for "contribution optimizing sales territory alignment." In contrast to previously suggested profit maximizing approaches, COSTA operates with sales response functions of any given concave form at the level of sales coverage units (SCUs) that cover a group of geographically demarcated individual accounts. Thus, COSTA works with sales response functions at a more aggregated level that requires less data than other profit maximization approaches. COSTA models sales as a function of selling time, which includes calling time as well as travel time, assuming a constant ratio of travel to calling time. In addition, the formulation of the model shows that an optimal solution requires only equal marginal profits of selling time across sales coverage units per territory, but not across SCUs of different territories.

Basically, COSTA consists of an allocation model and an assignment model, both of which are considered simultaneously. The allocation model optimally allocates the available selling time of a salesperson across the SCUs of his or her territory, whereas the assignment model assigns the SCUs to territories. Thus, COSTA predicts the corresponding profit contribution of every possible alignment solution, which enables one to perform "what-if"-analyses. The applicability of the model is supported by the development of a powerful heuristic solution procedure. A simulation study showed that COSTA provided solutions that are on average as close as 0.195% to an upper bound on the optimal solution. The proposed heuristic solution procedure enables one to solve large territory alignment problems because the computing time increases only quadratically with the number of SCUs and proportionally to the square root of the number of salespersons. In principle, we also show how COSTA might be expanded to solve the salesforce sizing as well as the salespersons' location problem.

The usefulness of COSTA is illustrated by an application. The results of this application indicated substantial profit improvements and also outlined the weaknesses of almost balanced territories. It is quite apparent that balancing is only possible at the expense of profit improvements and also does not lead to equal income opportunities for the salespersons. This aspect should be dealt with separately from territory considerations by using territory-specific quotas and linking variable payment to the achievement of these quotas. Furthermore, the superiority of COSTA turned out to be stable in a simulation study on the effect of misspecified sales response functions.

COSTA is of interest to researchers as well as practitioners in the salesforce area. It aims to revive the stream of research in the 1970s that already proposed sales territory alignment models aimed at maximizing profit. Such profit maximizing models are theoretically more appealing than approaches that strive to balance one or several attributes, such as potential or workload. COSTA's main advantage over previous profit maximizing approaches is that it is less complex. Consequently, COSTA demands less data so that even large problems can be solved close to optimality within reasonable computing times.
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1. Introduction

Companies often assign accounts exclusively to individual salespersons. Such an assignment supports the establishment of long-term relationships between accounts and salespersons, avoids competition among salespersons, offers better conditions for evaluation and control of the salesperson's performance, and increases the salesperson's morale and effectiveness (Albers 1989). Due to the opportunity costs of travel time, such an assignment is usually implemented by establishing sales territories (Churchill, Ford, and Walker 1997). To reduce the complexity of the sales territory design problem, companies generally use small geographical subareas—known as sales coverage units (SCUs)—instead of working with individual accounts. These SCUs frequently represent political districts or postal areas (Zoltners and Sinha 1983; Churchill, Ford, and Walker 1997). If the base locations of the salespersons have already been chosen, the problem is reduced to the alignment of sales territories, namely the optimal assignment of SCUs to sales territories covered by individual salespersons.

This problem is of considerable importance for companies for at least two reasons. First, there are significant sales and profit implications (Churchill, Ford, and Walker 1997). According to Zoltners and Sinha (1988), territory adjustments can raise sales anywhere from 2% to 7%. LaForge, Cravens, and Young (1986), who summarize previous empirical results, show similar improvements in comparable problems. Second, the decision of how to optimally align territories has to be made frequently because market conditions change quite often, and adjustments in salesforce size require realigned territories (Albers 1989).

1.1. Balancing Approach

Currently, the most popular approach to the alignment of territories is the balancing approach. It establishes territories that are as well balanced as possible with respect to one or more attributes (e.g., Zoltners and Sinha 1983; Churchill, Ford, and Walker 1997; Vandenbosch and Weinberg 1993). The most widely used attributes are potential and workload, whereas workload is usually measured by the number of sales calls. Equal potential or workload provides the salespersons with either equal output opportunities or equal input requirements and thus is considered to be fair (Churchill, Ford, and Walker 1997).

Unfortunately, the balancing approach does not guarantee territory alignments that maximize profit contribution. In addition, it does not provide information on the profit implications of territory designs. Thus, managers are neither informed on the possible profit improvements over existing territory designs nor can they evaluate the effect of modified base locations of salespersons and different salesforce sizes on profit.[1]

Moreover, the balancing approach often fails to reach its own goals of establishing territories with equal income opportunities and fair treatment of all salespersons. Apart from territory potential, many other factors influence sales, for example, territory size and intensity of competition (see Ryans and Weinberg (1979) and Albers (1989) for an overview). Hence, territories balanced with respect to potential usually do not lead to equal sales and, in case of equal commission rates, do not lead to equal income. However, compensation plans might consider such differences across territories by rewarding the achievement of territory-specific quotas (Albers 1996). Therefore, there is no further need for balanced territories. Equal workload may cause similar problems. Due to differing areal extents and traffic infrastructures of the territories, the required travel time for a certain amount of calling time often varies substantially. If workload is measured by selling time, the sum of calling and travel time, we have to take into account that travel times depend on the calling policy and the territory alignment. As a consequence, it is impossible to determine the workload without having settled the territories' boundaries first (Lodish 1975). Hence, the goal of fair treatment of all salespersons cannot be achieved.

1.2. Profit Maximization Approaches

In the 1970s, several authors proposed decision models for determining sales territory alignments that maximize profit contribution in order to overcome the

[1]Of course, if the managers additionally calibrate sales response functions to model the profit implications of balanced territories, the effect of modified base locations and different salesforce sizes might be evaluated as well. Yet, managers usually balance only territories and do not calibrate these functions such that the profit implications cannot be taken into account.

above-mentioned shortcomings of the balancing approach. (See Table 1 and the review by Vandenbosch and Weinberg (1993). Table 1 also includes the approach proposed in this paper.)

With the exception of Beswick and Cravens (1977), all these models were developed for simultaneously solving the optimal calling-time allocation across accounts and optimal assignment of accounts to salespersons (or territories). Shanker, Turner, and Zoltners (1975) and Zoltners (1976) simplify the solution of the problem by limiting the number of possible solutions beforehand, which allows the use of standard integer programming algorithms. However, either the prespecified number of possible solutions is too large, so that a solution cannot be found within an acceptable solution time, or the approximation of the original problem caused by the respective prespecification is unsatisfactory. In consequence, both approaches are not feasible for large problems (Zoltners and Sinha 1983).

Lodish (1975), on the other hand, proposes a heuristic solution procedure. First, he solves the problem of allocating selling time across accounts by assuming one super-territory in which the selling time is equal to the sum of selling times available to all salespersons. The formal structure of this problem is equivalent to that of the popular calling-time allocation model CALLPLAN (Lodish 1971) and can thus be solved with the corresponding heuristic procedure. Then, the decision maker has to reassign accounts step by step intuitively, so that the individual selling-time constraints are met and the marginal profit of selling time has become as equal as possible. Unfortunately, as we will demonstrate later (see § 2.3 and Appendix III), equal

Table 1 Comparison of Different Profit Maximization Approaches

Authors	Unit of Analysis	Required Specification	Solution Approach	Evaluation
Shanker, Turner, and Zoltners (1975)	Accounts	Calibration of S-shaped sales response functions Territory candidates	Set-partitioning problem	Impractical because the number of partitions increases exponentially with the number of accounts.
Zoltners (1976)	Accounts	Calibration of S-shaped sales response functions Discrete calling strategies	All-integer programming	Either the approximation through the calling strategies is too coarse or the CPU requirements are too time-consuming.
Lodish (1975)	Accounts	Calibration of S-shaped sales response functions	Two-step procedure 1. CALLPLAN 2. Manually equating marginal profit of selling time across territories	Manual procedure is rather slow. Goal of equating marginal profit of selling time across territories does not represent optimality conditions and thus does not lead to profit maximum.
Glaze and Weinberg (1979)	Accounts	Calibration of sales response functions	Alternating-step procedure 1. CALLPLAN 2. GEOLINE	Does not lead to profit maximum.
Beswick and Cravens (1977)	Sales coverage units (SCUs)	Calibration of concave sales response functions	Minimization of travel time s.t. balanced territories	Is similar to balancing approach and does not lead to profit maximum, no allocation of calls to individual accounts.
Skiera and Albers (1998)	Sales coverage units (SCUs)	Calibration of concave sales response functions	Maximization of profit by assigning SCUs to territories such that the resulting time allocation across SCUs leads to total profit optimum	Very close approximation of the profit maximum, solution is robust to misspecifications in the sales response function, no allocation of calls to individual accounts.

marginal profits of selling time are only optimal for the allocation of selling time across accounts or SCUs per territory, but not across territories. Glaze and Weinberg (1979) replaced only parts of Lodish's proposal by a procedure based on the balancing approach, so that they also do not find a profit contribution maximizing solution.

These models aim at solving the territory alignment problem by combining the well-developed calling-time allocation model CALLPLAN with a suitable assignment model. Hence, individual accounts (current and potential customers) have to be used as their unit of analysis. When facing large problems, this unit of analysis may require the calibration of thousands of response functions. This would create problems with dimensions too large to be efficiently solved. Therefore, it might be easier to work with aggregated response functions at the level of SCUs. These functions require less data and the corresponding models can be solved with simpler algorithms. Beswick and Cravens (1977) were the first to work with response functions where sales in an SCU depend upon calling time in that SCU. However, they also try to solve the assignment problem through the balancing approach and, therefore, do not maximize profit contribution.

1.3. Properties of the New Approach COSTA

Although proposed in the 1970s, profit maximizing approaches did not become the dominant method regarding sales territory alignment. Instead, in a later review by Zoltners and Sinha (1983), the balancing approach has been described as state-of-the-art. Despite its theoretical inferiority, it is very likely that the balancing approach has become the most popular method because it is easier to understand, requires only a moderate amount of data, and feasible solutions can even be produced by hand. To overcome the weaknesses of previous approaches, we developed a model called COSTA (contribution optimizing sales territory alignment). COSTA is both theoretically and practically more appealing than previous profit maximization approaches and the balancing approach. It determines simultaneously the optimal selling-time allocation across SCUs per territory as well as the optimal assignment of SCUs to a set of territories prespecified by the base locations of their salespersons.

Compared to previous profit maximization approaches, COSTA offers the following advantages (see Table 1): First, COSTA optimizes profit contribution without relying on the suboptimal assumption of equal marginal profit of selling time across SCUs of all territories (discussed in more detail in § 2.3). Second, COSTA works with aggregate sales response functions at the level of SCUs. This aggregation requires substantially less data and is also attractive for companies that cannot calibrate sales response functions at the level of accounts. Third, COSTA utilizes a new concept for incorporating travel time effects directly into the sales response function per SCU, depending on the assignment to a certain salesperson (i.e., territory). Fourth, COSTA is suitable for sales response functions of any given concave form. Fifth, the powerful algorithm developed for COSTA solves even large sales territory alignment problems within rather short computing time because the solution time for the algorithm increases only quadratically with the number of SCUs. Sixth, COSTA takes travel time as well as travel cost into consideration when solving the problem of allocating selling time across SCUs per territory.

In contrast to the balancing approach, COSTA searches for the alignment that maximizes profit contribution. In addition, it enables the comparison of the profitability of new territory designs with the already existing one. Furthermore, COSTA permits an assessment of the effects that modified base locations of salespersons have on profit contribution, as well as different salesforce sizes. However, COSTA does not give detailed recommendations for the allocation of individual calls. The advantages of COSTA and any other profit maximizing approach come at the cost of additional data needed to calibrate the sales response functions and at the risk of misspecifications in the sales response functions. Yet, as Lodish already pointed out in 1974, it is better to be vaguely right than to be precisely wrong. Another problem is that COSTA creates territories that are usually not comparable in terms of their income opportunities in the case of equal commission rates. Rather than balance them, it is more profitable to work with compensation plans that ensure equity among salespersons. It takes into account these individual differences across territories by linking compensation to the achievement of territory-specific sales quotas.

The remainder of the paper is organized as follows. In § 2, we will briefly discuss the basic idea of COSTA, introduce the new concept for taking travel time into consideration, outline the structure of the model, and describe alternative ways of estimating the sales response functions. A fast solution procedure for COSTA will be presented in § 3. Section 4 will illustrate an application of COSTA. The final section contains conclusions, managerial implications, and suggestions for future research.

2. Description of COSTA

2.1. Basic Idea

The basic idea of COSTA is to establish a relationship between a sales territory design and its profit contribution. Using an appropriate algorithm, it is then possible to determine the territory alignment that maximizes profit contribution. To establish such a relationship, COSTA works with sales response functions of any given concave form at the SCU level (e.g., multiplicative or modified exponential functions), linking the selling time of a given salesperson to profit contribution. Mantrala, Sinha, and Zoltners (1992) show that the optimal deployment of resources to accounts within the SCU will always result in concave functions, even if the individual account functions are not concave. Estimating the sales response function at the SCU level, rather than at the level of individual accounts, offers the advantage that less data is required.

$$S_{j,r} = f_{j,r}\,(t_{j,r,\mathrm{call}}) \qquad (j \in J,\ r \in R), \quad (1)$$

where:

$S_{j,r}$: sales in the rth SCU if assigned to the jth salesperson,

J: index set of territories represented by the base locations of the salespersons,

R: index set of sales coverage units (SCUs),

$f_{j,r}(t_{j,r,\mathrm{call}})$: sales response function providing sales in the rth SCU if the jth salesperson devotes a time of $t_{j,r,\mathrm{call}}$ to calling.

Like other authors (Mantrala, Sinha, and Zoltners 1992, 1994), we assume that sales in the SCUs are mutually independent. The sales response functions might differ across SCUs and salespersons. Thus, we can model the effect of varying characteristics across SCUs and salespersons so that we can take into account, for example, the effect of varying selling abilities across salespersons or the negative effect of disrupting an existing relationship between accounts (i.e., SCUs) and their salesperson. We solve both an allocation and an assignment problem simultaneously with the objective of maximizing profit contribution by using these sales response functions. The allocation problem asks how to allocate the selling time across SCUs assigned to a given territory, while the assignment problem addresses the question of how to assign the SCUs to territories served by individual salespersons.

2.2. Consideration of Travel Times

Models working with sales response functions on the account level considered travel time in the following way: They used only calling time in their sales response functions at the account level and calculated the required travel time on the assumption that the number of trips to an SCU is equal to the highest number of calls to one of the accounts in that SCU (Lodish 1971 and his model CALLPLAN). We have no specific information on the number of calls to individual accounts because our sales response functions are based on the SCU level. Thus, we assume that the sales response function represents sales with selling time optimally allocated across accounts. In addition, we propose to express the travel time ($t_{j,r,\mathrm{travel}}$) that is necessary for carrying out the calls as a constant multiplier $q_{j,r}$ of the calling time ($t_{j,r,\mathrm{call}}$) the jth salesperson spends in the rth SCU.

$$t_{j,r,\mathrm{travel}} = q_{j,r} \cdot t_{j,r,\mathrm{call}} \qquad (j \in J,\ r \in R). \quad (2)$$

This multiplier can be calculated as follows: We start with the information about the average duration $\mathrm{TT}_{j,r}$ for a trip from the jth salesperson's base location to the rth SCU, which frequently corresponds to the normal daily working time. On each trip, the salesperson starts from his or her base location, calls on a number of accounts in a SCU, and returns afterwards to his or her base location. Depending on the number of calls per trip, we split the average duration of a trip into the time available for calling and the time required for traveling. Data pertaining to the travel time of an average round trip $RT_{j,r}$ of the jth salesperson from his or her base location to the rth SCU can be gathered from software products such as DISTANCE or AutoRoute.[2] Results of a survey described in Skiera (1996) and our own experience indicate that information on the average calling times $CD_{j,r}$ (including possible waiting times) per account in the rth SCU and average travel times $SD_{j,r}$ required to get from one account to the next within the rth SCU are acquired quite easily by means of subjective estimation by the sales manager.[3] Now, if $n_{j,r}$ denotes the average number of calls on one trip from the jth salesperson's base location to the rth SCU, the average duration $TT_{j,r}$ of one trip into the rth SCU by the jth salesperson can be formally separated as follows:

$$TT_{j,r} = RT_{j,r} + (n_{j,r} - 1) \cdot SD_{j,r} + n_{j,r} \cdot CD_{j,r} \quad (j \in J, r \in R) \quad (3)$$

By solving Equation (3) for $n_{j,r}$, we get the average number of accounts ($n_{j,r}$) that can be called on a trip:

$$n_{j,r} = \frac{TT_{j,r} - RT_{j,r} + SD_{j,r}}{CD_{j,r} + SD_{j,r}} \quad (j \in J, r \in R). \quad (4)$$

The sales manager has to provide data for all variables on the right-hand side of Equation (4). The consideration of overnight stays is possible by specifying $TT_{j,r}$ as a multiple of the normal daily working time. In this case, we assume that management orders its salespersons to stay overnight on each trip to the rth SCU. In the case that managers feel uncomfortable with noninteger values of $n_{j,r}$, these values may be rounded to the next integer. In addition, we might limit the number of calls per tour ($n_{j,r}$) to any specific number, for example, the number of accounts available in the SCU, or set $n_{j,r}$ to any prespecified number. After having determined $n_{j,r}$, we are now able to derive the multiplier $q_{j,r}$ as the ratio of travel time to calling time:

$$q_{j,r} = \frac{RT_{j,r} + (n_{j,r} - 1) \cdot SD_{j,r}}{n_{j,r} \cdot CD_{j,r}} \quad (j \in J, r \in R). \quad (5)$$

Assuming that this ratio $q_{j,r}$ remains constant for all levels of selling time $t_{j,r}$, we are able to work with selling time as the only decision variable in the allocation problem of our model:

$$t_{j,r,\text{call}} = \frac{1}{1 + q_{j,r}} \cdot t_{j,r} \quad (j \in J, r \in R), \quad (6)$$

$$t_{j,r,\text{travel}} = \frac{q_{j,r}}{1 + q_{j,r}} \cdot t_{j,r} \quad (j \in J, r \in R). \quad (7)$$

The simplicity of our model results from the assumption of a constant ratio $q_{j,r}$. This assumption is a reasonable one if the following three conditions are fulfilled:

(a) The average travel times $SD_{j,r}$ and the average calling times $CD_{j,r}$ remain constant for all levels of selling time $t_{j,r}$. Varying average travel times $SD_{j,r}$ might occur if closely dispersed accounts are called on for lower levels of selling time $t_{j,r}$ and widely dispersed accounts are called on for higher levels of selling time $t_{j,r}$ (or vice versa). The average calling times $CD_{j,r}$ might not remain constant if accounts that require a long call duration are called on for lower levels of selling time $t_{j,r}$ and accounts that require shorter call durations are called on for higher levels of selling time $t_{j,r}$. In both situations, Condition (a) is not fulfilled. Yet, such situations are unlikely to occur in practice and are very difficult to be accounted for by even more sophisticated models that consider sales at the level of individual accounts.

(b) The error due to selling times that imply a noninteger number of trips is small. This error occurs when the chosen selling time $t_{j,r}$ implies a noninteger number of trips $y_{j,r} = t_{j,r}/TT_{j,r}$ of the jth salesperson to the rth SCU. In this situation, the salesperson is forced to combine trips to adjacent SCUs. This is not a serious problem as long as the selling time of a salesperson is defined for a rather long period (e.g., for a year). In case of 200 working days and 10 SCUs per territory, the average number of trips to one of these SCUs is 20. Implying, for example, 20.5 trips would mean that we incorrectly predicted the travel time between accounts for 0.5 trips out of 20 trips. That means

[2]DISTANCE is supplied by ptv, Germany, and AutoRoute by NextBase, England.

[3]For more sophisticated methods of estimating the average travel time to the next customer, see Rosenfield, Engelstein, and Feigenbaum (1992).

we incorrectly predicted the travel time in 2.5% of the cases. Given that we underestimated the travel time by 20% and the travel times between accounts represent 20% of the total selling time, our approximation resulted only in an error of 2.5%*20%*20% = 0.1%. That means we predicted that 0.1% of the selling time was used for calling when it was actually used for traveling.

(c) The number of calls to any one of the individual account is not higher than the number of trips to that SCU. This condition is violated if the share of calls to the account with the highest number of calls with respect to the total number of calls in that SCU is higher than $1/n_{j,r}$. In the case of a pharmaceutical company whose salespersons perform eight calls per trip, this situation means that none of the accounts will receive more than 12.5% of all calls in the considered SCU. That percentage is even higher for fewer calls per day. If such a rather unlikely situation occurs, we recommend that adjacent SCUs are grouped into a newer, larger SCU so that the percentage of calls to the account with the highest number of calls meets the above condition.

This discussion shows that the assumption of a constant ratio of travel time to calling time is plausible and allows for an easy incorporation of travel time. Of course, a detailed formulation of the whole problem as a simultaneous territory alignment, calling-time allocation, call-scheduling and trip-planning problem would reflect reality even more precisely. However, no one has been able to efficiently solve the resulting complex decision model to date.

2.3. Modeling the Allocation and Assignment Problem

We determine the optimal sales territory alignment by solving the following nonlinear mixed-integer Model (8)–(12) that integrates the allocation and the assignment problem:

$$\sum_{j\in J}\sum_{r\in R}\Bigg[\underbrace{g_r\cdot f_{j,r}\left(\frac{1}{1+q_{j,r}}\cdot t_{j,r}\right)}_{\text{profit contribution of calling time}} - \underbrace{h_{j,r}\left(\frac{q_{j,r}}{1+q_{j,r}}\cdot t_{j,r}\right)}_{\text{travel costs}}\Bigg]\cdot x_{j,r}\rightarrow \max! \tag{8}$$

$$\sum_{r\in R} t_{j,r} \le T_j \qquad (j\in J), \tag{9}$$

$$t_{j,r}\ge 0 \qquad (j\in J,\ r\in R), \tag{10}$$

$$\sum_{j\in J} x_{j,r} = 1 \qquad (r\in R), \tag{11}$$

$$x_{j,r}\in\{0,1\} \qquad (j\in J,\ r\in R), \tag{12}$$

where

g_r: gross margin of sales in the rth SCU,

$f_{j,r}\,(1/(1+q_{j,r})\cdot t_{j,r})$: sales as a function of the calling time (expressed as a fraction of selling time $t_{j,r}$) for the rth SCU if assigned to the jth salesperson,

$h_{j,r}\,(q_{j,r}/(1+q_{j,r})\cdot t_{j,r})$: travel cost as a function of travel time (expressed as a fraction of selling time $t_{j,r}$) for the rth SCU if assigned to the jth salesperson,

J: index set of territories represented by the base locations of the salespersons,

R: index set of sales coverage units (SCU), $x_{j,r}$ = {1 if the rth SCU is assigned to the jth salesperson, 0 else,

$t_{j,r}$: selling time available for calling in and traveling to and in the rth SCU if assigned to the jth salesperson,

T_j: maximum selling time available to the jth salesperson.

The problem is finding the values for the binary assignment variables $x_{j,r}$ and the continuous selling-times variables $t_{j,r}$, so that total profit contribution in the objective function (8) is maximized. In more detail, this includes profit contribution (sales multiplied by average gross margin) resulting from calling time minus travel cost (expressed as a function of travel time as discussed in (7)). This term is multiplied by the binary assignment variable $x_{j,r}$ and summed over all SCUs and salespersons. Constraints (9) and (10) ensure that the individual selling-time constraints of all salespersons are met and that selling times in all SCUs are positive. Constraint (11) and the binary definition of $x_{j,r}$ in (12) provide for the exclusive assignment of an SCU to exactly one salesperson.

The structure of our model also demonstrates why the optimal solution does not necessarily possess the property of equal marginal profits of selling time across all SCUs. This property would only be fulfilled if we had just one constraint (9) for the total selling time and, thus, only one Lagrange multiplier. However, we have separate selling-time constraints (9) for each territory and also exclusivity constraints (11) for each SCU. This results in different Lagrange multipliers and, hence, unequal marginal profits of selling time across territories (also see Appendix III).

In contrast to most proposed sales territory alignment models, Model (8)–(12) no longer contains a constraint to ensure the contiguity of the sales territories. As long as all profit drivers are taken into account appropriately, maximizing profit contribution will provide solutions with contiguous or almost contiguous sales territories automatically.[4] The explicit consideration of a contiguity constraint is necessary only in models for the balancing approach where tight intervals for the balancing attributes can result in completely noncontiguous solutions. However, in case a company does not want to tolerate noncontiguous territories for reasons that are not elaborated within Model (8)–(12), for example, transparency of the territories, the following contiguity constraint might be added to Model (8)–(12):

$$\sum_{w \in W_{j,r}} \prod_{\tau \in R_w} x_{j,\tau} \geq 1 \qquad (j \in J,\ r \in R), \quad (13)$$

where

R_w: set of SCUs belonging to the wth path,

$W_{j,r}$: set of paths between the base location of the jth salesperson and the rth SCU.

In addition to establishing a profit maximizing alignment, Model (8)–(12) as well as Model (8)–(13) provide the necessary structure to evaluate the effects of different base locations and different salesforce sizes. To examine the first effect, we simply modify the salespersons' locations, determine the new calling- and travel-time fractions of selling times by using Equations (6) and (7) for the respective salespersons, and solve the respective problem again. The effects of different salesforce sizes are assessed by modifying the size of the index set of salespersons, by adding or deleting salespersons, and providing locations for those who were added (Zoltners 1981). Certainly, applying nonlinear search techniques like the golden-section search (Himmelblau 1972) over the number of salespersons tends to make the solution of these problems more efficient. We agree with Zoltners (1981) that location problems in practice are usually restricted to such an extent that "what-if analyses" provide reasonably good solutions.

[4]The authors are indebted to one of the reviewers who encouraged them to elaborate more on that point.

2.4. Estimation of the Sales Response Function

We suggest two different approaches that differ in the required amount of data and statistical expertise to estimate the sales response functions. If there is enough data available, we recommend estimating a sales response function statistically by using the following as independent variables: area characteristics of the SCU (e.g., potential, marketing expenditure of the company, competitive pressure), personal characteristics of the salesperson (e.g., selling experience of the salesperson), and calling time (expressed here as a fraction of selling time). An example is the following multiplicative sales response functions:

$$S_{j,r} = f_{j,r}\left(\frac{1}{1+q_{j,r}} \cdot t_{j,r}\right) = \alpha \cdot \left(\prod_{k \in K} v_{k,r}^{\beta_k}\right) \cdot \left(\prod_{k' \in K'} w_{k',j}^{\beta_{k'}}\right) \cdot \left(\frac{1}{1+q_{j,r}} \cdot t_{j,r}\right)^{b_r} \quad (j \in J,\ r \in R), \quad (14)$$

where

α: scaling constant,

b_r: elasticity of sales with respect to calling time in the rth SCU,

$\beta_k, \beta_{k'}$: corresponding elasticity of sales with respect to the kth area (k'th personal) characteristic,

K, K': index set of the area (personal) characteristics,

$q_{j,r}$: ratio of travel time to calling time of the jth salesperson in the rth SCU,

$t_{j,r}$: selling time of the jth salesperson in the rth SCU,

$v_{k,r}$: value of the kth area characteristic in the rth SCU,

$w_{k',j}$: value of the k'th personal characteristic of the jth salesperson.

Several studies have shown that such an estimation yields valid sales response functions (see overviews in Ryans and Weinberg 1979, Albers 1989), and that the parameters of these functions are stable over time (Ryans and Weinberg 1987). Furthermore, our SCU-response function in Equation (14) incorporates the effect of different travel times by assuming a constant calling-time fraction of selling time. Note that response function (14) has indices for j (salesperson influence) and r (SCU influence). The area characteristics $v_{k,r}$ allow, for example, to model the effect of a different number of accounts in a SCU, whereas the personal characteristics $w_{k',j}$ incorporate the effect of different selling abilities or selling experiences with a specific SCU. The latter also enables one to model the effect of disrupting a relationship between a salesperson and his or her accounts.

In case a company encounters a lack of either data or statistical expertise, we recommend calibrating the response functions subjectively. This task is suitable because the parameters of the sales response functions of a particular SCU differ only with respect to personal characteristics of the assigned salesperson (incorporated in Equation (14) by the variables $w_{k',j}$). These differences come from only three sources (Skiera 1996). First, and probably most important, travel times for serving the particular SCU differ across base locations of salespersons. Second and third, salespersons might possess different selling abilities and different experiences with respect to the accounts in the particular SCU. The first difference is modeled by considering travel times as outlined above. The second one might be incorporated by using a procedure for eliciting subjective estimates similar to the one proposed by Lodish (1976). The third one can be considered by asking the sales manager for the additional percentage of calling time that an inexperienced salesperson needs to become as familiar with the accounts within the SCU as the other salesperson is.

3. Solution Procedure and Software Impementation of COSTA

3.1. Algorithm

The algorithm proposed for solving Model (8)–(12) builds upon the idea that the problem is closer to an allocation than to a combinatorial problem. This characteristic has enabled us to come up with a powerful solution procedure based mainly on marginal analysis and a backward deletion of assignments. The procedure is also flexible enough to work with any given concave sales response function. The computing time of this algorithm increases only quadratically in the number of SCUs. If contiguity is still required, we simply delete the noncontiguous assignments of SCUs to salespersons, reimplement some suitable previously deleted assignments, and apply the backward deletion algorithm again. Although the allocation and the assignment problem are solved simultaneously, the algorithm for both problems will be presented separately to facilitate the presentation.

3.1.1. Algorithm for the Allocation Problem. For a given assignment the following allocation problem remains:

$$\pi_j = \sum_{r \in R_j} \mathrm{PC}_{j,r} = \sum_{r \in R_j} \left[g_r \cdot f_{j,r} \left(\frac{1}{1 + q_{j,r}} \cdot t_{j,r} \right) - h_{j,r} \left(\frac{q_{j,r}}{1 + q_{j,r}} \cdot t_{j,r} \right) \right] \rightarrow \max! \qquad (j \in J), \quad (15)$$

$$\sum_{r \in R_j} t_{j,r} \le T_j \qquad (j \in J), \quad (16)$$

$$t_{j,r} \ge 0 \qquad (j \in J,\ r \in R_j), \quad (17)$$

where

$\mathrm{PC}_{j,r}$: profit contribution of the jth salesperson in the rth SCU,

R_j: index set of SCUs assigned to the jth salesperson,

T_j: maximum selling time available to the jth salesperson.

Objective (15) represents the sum of profit contributions in the SCUs of the jth salesperson subject to positive selling times in all SCUs (constraint (17)) that are in sum smaller than the total amount of selling time available to the salesperson (constraint (16)). Travel cost—not the opportunity cost of time—is usually small compared to profit contribution from sales, so that it is profitable to allocate the available selling time completely. A solution that does not fully utilize the total selling time is not realistic. However, we check whether the optimal solution can be improved by reducing the total allocated time and we provide information to the user in such a case. Consequently, constraint (16) becomes an equation, and the optimal solution for Model (15)–(17) possesses the following property (see Albers 1997 and Appendix III):

$$t_{j,r,\mathrm{opt}} = \frac{g_r \cdot \epsilon_{j,r,\mathrm{opt}} \cdot S_{j,r,\mathrm{opt}} - \gamma_{j,r,\mathrm{opt}} \cdot C_{j,r,\mathrm{opt}}}{\sum_{\tau \in R_j} (g_\tau \cdot \epsilon_{j,\tau,\mathrm{opt}} \cdot S_{j,\tau,\mathrm{opt}} - \gamma_{j,\tau,\mathrm{opt}} \cdot C_{j,\tau,\mathrm{opt}})} \cdot T_j \quad (j \in J,\ r \in R_j), \quad (18)$$

where

$\epsilon_{j,r,\mathrm{opt}}$: elasticity of sales in the rth SCU when assigned to the jth salesperson with respect to selling time (holding at the optimum),

$\gamma_{j,r,\mathrm{opt}}$: elasticity of travel cost for the rth SCU when assigned to the jth salesperson with respect to selling time (holding at the optimum),

$S_{j,r,\mathrm{opt}}$: optimal sales volume in the rth SCU if assigned to the jth salesperson,

$C_{j,r,\mathrm{opt}}$: optimal travel cost in the rth SCU if assigned to the jth salesperson.

Based on this optimality condition, we improve a starting solution by iteratively inserting current values for $S_{j,r,\mathrm{opt}}$, $\epsilon_{j,r,\mathrm{opt}}$, $C_{j,r,\mathrm{opt}}$, and $\gamma_{j,r,\mathrm{opt}}$ into Equation (18) until the solution is sufficiently close to the optimum (Albers 1997). This procedure is especially attractive because it enables one to determine optimal solutions for any given concave sales response function and convex or linear travel cost function (for a detailed description of the algorithm, see Appendix I).

3.1.2. Algorithm for the Assignment Problem. With respect to the assignment problem, we use a backward deletion algorithm that starts with the infeasible solution of all SCUs being assigned to all salespersons. This solution is the optimal one if the exclusivity constraint (11) is relaxed. Then, the algorithm deletes assignments step by step until the exclusivity constraint (11) is fulfilled. In each step, the algorithm takes that assignment of a SCU to a salesperson that leads to the smallest loss in profit contribution. The loss $L_{j,r}$ is equal to the profit contribution of the current assignment $\mathrm{PC}_{j,r}$ minus the additional profit that might be gained by using the selling time of the deleted assignment for the allocation to other SCUs of the salesperson. The latter is approximated by multiplying the mean marginal profit contribution rate MMPC_j with the amount of deleted selling time $t_{j,r}$:

$$L_{j,r} = \mathrm{PC}_{j,r} - t_{j,r} \cdot \mathrm{MMPC}_j \quad (j \in J,\ r \in R). \quad (19)$$

Afterwards we look for the rth SCU that, in case of an assignment to the jth salesperson, provides the smallest regret. This regret is defined by the maximum difference between the maximal and minimal profit contribution loss across the salespersons to which the rth SCU is currently assigned. If we delete these assignments step by step, we delete the most inappropriate assignments early on, so that the other, more attractive assignments of that particular SCU are still possible. We optimally reallocate the salesperson's selling time across his (nondeleted) SCUs before deleting the next assignment. Consequently, we very gradually approach the final and feasible solution with the exclusive assignments of all SCUs to exactly one salesperson.

This procedure is repeated until all SCUs are assigned to exactly one salesperson. If computational time is not important, the backward deletion algorithm could be repeated again after having determined a new starting solution by reimplementing some suitable previously deleted assignments. According to our experience, this repetition rarely leads to improved solutions. If the solution exhibits a few noncontiguous assignments and management insists on strict contiguity, then we simply delete noncontiguous assignments, reimplement some suitable assignments, and apply the backward deletion algorithm again (for a more detailed description of this assignment procedure, see Appendix II).

3.1.3. Salesforce Sizing and Location Analysis. In addition, Model (8)–(12) provides the structure to solve the salesforce sizing problem as well as the salespersons' location problem. Although the model complexity and hence the computing effort might increase dramatically by solving the four problems of salesforce sizing, salespersons' location, sales territory alignment, and selling-time allocation simultaneously, our solution procedure might easily be implemented in sequential or hierarchical solution procedures like those that have been proposed by Hess and Samuels (1971), Beswick and Cravens (1977), Glaze and Weinberg (1979), and Drexl and Haase (1996).

3.1.4. Computing Time for and Solution Quality of the Algorithm. This algorithm finds a solution for problems of 10 salespersons and 50 SCUs within less than one second on a PC with Pentium-133 processor.

Even larger problems can be solved within reasonable computing time. For example, the solution of a problem with 100 salespersons and 2,000 SCUs, which represents the size of a typical problem for large pharmaceutical companies in Germany, takes approximately 100 minutes. The computing times for problems with simulated data and different sizes are presented in Table 2. Note that these computing times are very conservative. They can be decreased by considering only those assignments of our assignment procedure in Step 1 that have a calling-time fraction of selling time above a certain threshold value.

We have taken these data to run a regression analysis with computing time CTIME as the dependent variable and the number of SCUs $|R|$ and the number of salespersons $|J|$ as independent variables. This resulting equation explains 99.6% of the variance:

$$\mathrm{CTIME} = 0.0001126 \cdot |R|^{1.98} \cdot |J|^{0.54}. \quad (20)$$

The parameter values are statistically highly significant. The result is very encouraging because the computing time increases only quadratically with the number of SCUs and proportionally to the square root of the number of salespersons. The result implies that even extremely large problems can be solved with this algorithm.

Our algorithm represents only a heuristic solution procedure. Its quality can only be evaluated against an upper bound on the value of the objective function. Haase (1997) has proposed a linear programming formulation of the special problem structure described in § 2.3 on the basis of a piecewise linear envelope approximation of the response functions and a relaxation of the exclusivity constraint. The optimal solution of that problem provides an upper bound for the solution of our problem. Our algorithm deviates only 0.195% on average with a standard deviation of 0.202% from these upper bound solutions for a set of problems comparable to those in Table 2. These results support the strength of our algorithm.

Table 2 Computing Time for Different Problem Sizes

Number of Territories $\|J\|$	Number of SCUs $\|R\|$	Ratio: $\frac{\|R\|}{\|J\|}$	Computing Time (in sec.)
10	50	5	1
10	100	10	4
10	200	20	15
10	400	40	49
15	75	5	3
15	150	10	10
15	300	20	41
15	600	40	141
25	125	5	8
25	250	10	31
25	500	20	132
25	1,000	40	502
50	250	5	40
50	500	10	161
50	1,000	20	933
50	2,000	40	3,093
100	500	5	290
100	1,000	10	1,359
100	2,000	20	5,951

3.2. Software Implementation

The use of decision models depends heavily on the associated software implementation. To provide a program that can be used easily, we have developed a software named COSTA™. This software displays the shape of the sales territories in map form to the sales manager and allows him or her to either align the territories on the map interactively or start the optimization procedure described in § 3.1. The solution procedure itself is programmed in C++. The results and a comparison of different territory alignments are presented either in graphical or tabular form. In addition, COSTA™ enables sales managers to conveniently exchange data with their spreadsheet applications.

4. Application of COSTA

This section presents the results of an application of COSTA. In § 4.1 we illustrate the usefulness of the information that sales managers obtain from applying COSTA. In § 4.2 the sales territory alignment derived from COSTA is compared with that of the balancing approach. An analysis of the marginal profits of time in the different territory alignments can be found in § 4.3. The influence of estimation errors in the sales response functions is determined in § 4.4.

4.1. Results of the Application

COSTA was applied in a mid-sized German company that had traditionally served the market almost exclusively by mail order, but planned to build up a salesforce to better serve the larger accounts of its target group. The company had hired 10 salespersons within the last few months and adopted the 95 two-digit postal areas as SCUs. Territories were set up as shown in Figure 1, in which the numbers of territories mark the salespersons' base locations. The company intended to have the two SCUs be covered by their back office personnel in the northernmost part of Germany (Territory 11) and in the vicinity of their headquarters. In addition, for internal reasons it was decided that the size of Territory 10 should remain unchanged.

The company assumed that no substantial performance differences between their salespersons existed and believed that it was not close to a saturation level. Because the company did not have any response data, the following multiplicative sales response function was derived from subjective judgments by the management:

$$S_{j,r} = \alpha \cdot \mathrm{POT}_r^{\eta} \cdot \left(\frac{1}{1 + q_{j,r}} \cdot t_{j,r}\right)^b = 1350 \cdot \mathrm{POT}_r^{0.625} \cdot \left(\frac{1}{1 + q_{j,r}}\right)^{0.375} \cdot t_{j,r}^{0.375} \quad (j \in J,\ r \in R), \quad (21)$$

where POT_r was the number of potential accounts. The ratio of travel time to calling time $q_{j,r}$ differs across salespersons and the potential POT_r differs across SCUs. In consequence, these sales response functions are salesperson- as well as SCU-specific. In this application, the company ordered its salespeople to stay overnight for one night in certain SCUs if this SCU could be served more efficiently this way. However, to avoid overnight stays as much as possible, the company asked for contiguous territories.

These sales response functions and further information on the calculation of the calling-time fraction of selling time and travel cost[5] enabled us to predict

[5]The data of the application discussed here are provided as an EXCEL-spreadsheet on our home page in the Internet (http://halfrunt.bwl.uni-kiel.de:80/bwlinstitute/Marketing/).

Figure 1 **Territory alignments**

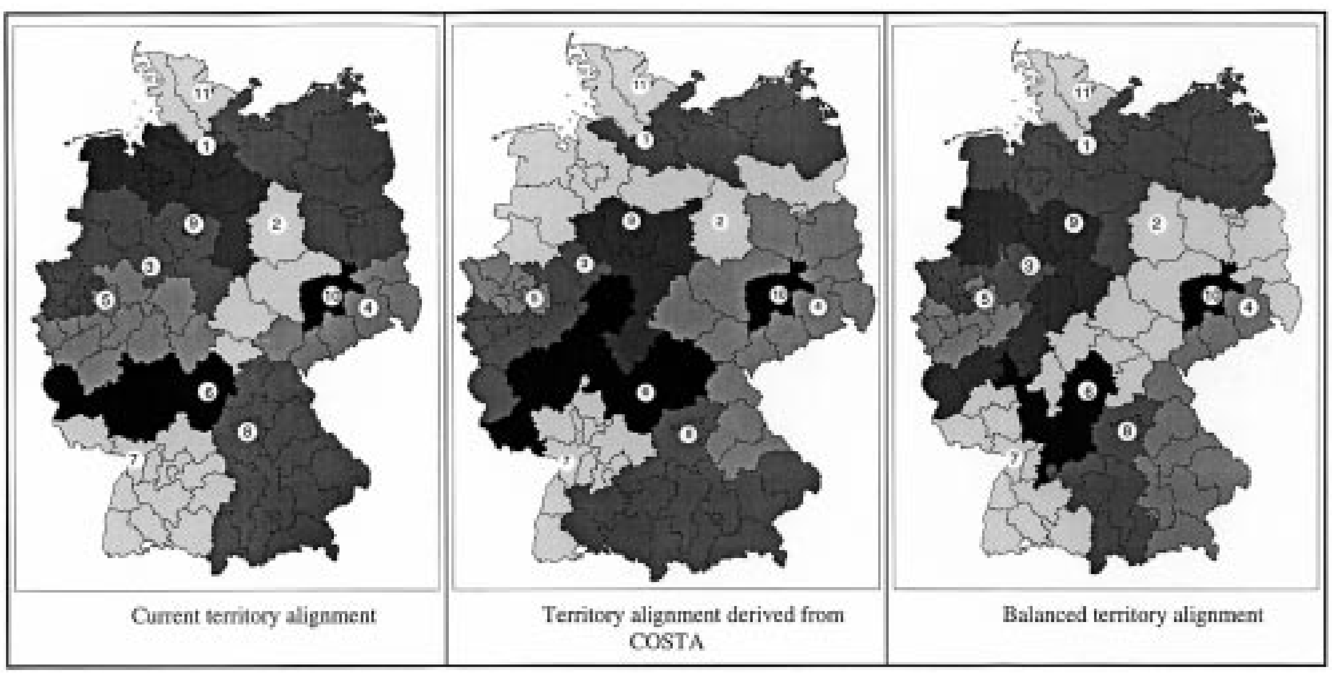

Table 3 Characteristics of the Current Territory Alignment

Territory	Sales (in DM)	Profit Contribution After Selling Expenses (in DM)	Potential (Number of Accounts)	Number of Calls	Marginal Profit of Selling Time
1	2,090,504	510,042	2,543	715	537
2	841,457	73,784	604	710	216
3	2,898,635	799,294	4,394	668	745
4	1,184,999	198,798	950	794	304
5	2,470,860	651,040	3,232	728	635
6	2,385,648	609,833	3,164	689	613
7	2,912,532	807,281	4,242	715	748
8	2,711,837	726,205	3,723	726	697
9	1,627,236	354,816	1,637	743	418
10	809,060	120,414	448	973	208
11	—	—	385	—	—
Total	19,932,768	4,851,508	25,322	7,462	—

sales and profit contribution for the current territory alignment (Table 3). Except for Territory 10, sales and profit contribution were highest in Territories 3, 7, and 8 and lowest in Territories 2 and 4. In total, the salespersons were able to make 7,462 calls. Salesperson 4 made the highest number of sales calls, but realized only low sales and profit contribution. This is due to the fact that this salesperson had relatively few accounts to call on.

One major shortcoming of the current alignment is that the northwestern territories (especially Territories 2 and 4) are substantially smaller than the territories in the south. To some extent this deficiency was reduced by the territory alignment derived from COSTA (see Figure 1). In that solution, most territories expanded toward the southeast and Territories 2 and 4 were enlarged. Yet, Territories 1, 2, 4, and 9 still had a small number of relevant accounts, which indicates that there are too many salespersons in the northeast. In contrast, Salespersons 7 and 8 had territories with large numbers of relevant accounts. They were both located in areas with high numbers of accounts. The upper part of Table 4 indicates that the territory alignment derived from COSTA increased profit contribution and sales by 6.1% and 4.5%, respectively. Note that the absolute increase in profit contribution of 294,864 DM well exceeds the cost of one of the 10 salespersons.

4.2. Results Compared to Those of the Balancing Approach

To emphasize the relevance of our critique of the balancing approach, we have also established a territory alignment that is almost balanced with respect to the number of relevant accounts (see Figure 1), using an algorithm developed by Skiera and Jordan (1996). All territories in this alignment have a potential within $\pm 5\%$ of the average potential per territory (see Table 4). Territories 10 and 11 are exceptions due to the reasons mentioned above. This alignment yields a somewhat higher profit contribution than the current territory alignment (compare Tables 3 and 4). However, it leads to a profit contribution that is 3.8% lower than those of the territory alignment derived from COSTA. The reason for this is that the losses in profit contribution in Territories 7 and 8 are not compensated by the gains in Territories 1, 2, and 9. Both sales and profit contribution in all territories (except for Territories 10 and 11) are more balanced than in the other territory alignments. But as sales and profit contribution in Territory 1 are still 17.3% and 29.1% higher than in Territory 4, we conclude that equal sales, and hence equal income opportunities, are not completely achieved in this territory alignment. These results underline the relevance of our critique of the balancing approach as it neither establishes profit maximizing sales territories nor leads to equal income opportunities. Hence, we

Table 4 Characteristics of Territory Alignment Solutions as Derived from COSTA and the Balancing Approach

Territory	Sales (in DM)	Profit Contribution After Selling Expenses (in DM)	Potential (Number of Accounts)	Number of Calls	Marginal Profit of Selling Time
		Results for COSTA			
1	1,699,544	380,811	1,660	826	437
2	1,953,836	452,069	2,257	700	502
3	2,203,248	544,974	2,595	764	566
4	1,943,315	456,676	2,251	706	499
5	2,408,677	632,829	2,836	830	619
6	2,308,201	580,690	2,882	730	593
7	2,863,030	791,281	3,961	758	735
8	3,081,392	853,954	4,583	721	791
9	1,566,757	332,675	1,464	800	402
10	809,060	120,414	448	973	208
11	—	—	385	—	—
Total	20,837,060	5,146,372	25,322	7,808	—
		Results for balanced territories			
1	2,311,993	593,515	2,858	757	594
2	2,085,096	500,916	2,710	632	536
3	2,149,344	535,669	2,704	685	552
4	1,970,980	459,684	2,740	565	506
5	2,241,384	572,599	2,644	776	576
6	2,151,416	532,108	2,698	685	553
7	2,232,555	566,681	2,740	735	573
8	2,220,779	557,010	2,755	713	570
9	2,113,485	519,425	2,640	683	543
10	809,060	120,414	448	973	208
11	—	—	385	—	—
Total	20,286,092	4,958,021	25,322	7,203	—

Table 5 Superiority of COSTA's Solution (derived for $\eta = 0.625$ and $b = 0.375$) to the Solution of the Balancing Approach for Different Combinations of True Parameter values η and b in the Sales Response Function

	b								
η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.21%	2.49%	3.72%	4.88%	5.89%	6.63%	6.79%	5.59%	1.67%
0.2	1.16%	2.39%	3.57%	4.70%	5.68%	6.44%	6.65%	5.48%	1.68%
0.3	1.10%	2.28%	3.42%	4.54%	5.52%	6.28%	6.51%	5.42%	1.80%
0.4	1.05%	2.18%	3.31%	4.37%	5.35%	6.12%	6.42%	5.42%	2.02%
0.5	1.00%	2.09%	3.17%	4.23%	5.20%	5.99%	6.32%	5.43%	2.29%
0.6	0.95%	2.01%	3.06%	4.08%	5.05%	5.85%	6.21%	5.45%	2.58%
0.7	0.90%	1.93%	2.95%	3.95%	4.90%	5.70%	6.08%	5.46%	2.87%
0.8	0.87%	1.84%	2.82%	3.82%	4.76%	5.57%	5.96%	5.41%	3.14%
0.9	0.82%	1.76%	2.72%	3.69%	4.60%	5.40%	5.81%	5.33%	3.35%

feel that companies might be better served by using COSTA to establish profit maximizing sales territory alignments.

4.3. Marginal Profits of Selling Time in the Different Territory Alignments

Another very important result in Table 4 is that marginal profits of selling time differ among salespersons in the solution derived from COSTA.[6] Apart from Territory 10, which remains unchanged, Salesperson 9, followed by Salespersons 1, 4, and 2, has the lowest marginal profit of selling time while the value for Salesperson 8 is almost double that of Salesperson 1. The reason for the rather small marginal profit of selling time in the northwestern territories is the limited sales opportunities in that area, whereas, Salesperson 8, for example, is located in the south of Germany (Bavaria) where more accounts are at his disposal, and, hence, higher sales opportunities are given. This information highlights the shortage of salespersons in the southwest (or, to put it negatively, the abundance of salespersons in the northeast). The discrepancy in the marginal profits of selling time for all salespersons decreases in the territory alignment of the balancing approach. However, the disadvantage of this decrease is a loss in profit.

4.4. Sensitivity Analysis of the Superiority of COSTA's Solution to Estimation Errors in the Sales Response Function

COSTA's solution predicts a 3.8% higher profit contribution than the balancing approach's solution. Yet, an optimal solution of COSTA is always superior to an optimal solution of the balancing approach if both solutions are compared to the profit contribution calculated upon the sales response function used by COSTA. Therefore, one may suspect how profit contribution improvements might change if the calibration of the sales response functions contain some estimation error. Therefore, we tested in Table 5 the sensitivity of the superiority of COSTA's solution to that of the balancing approach. We took the assignment from the solution of COSTA derived for $\eta = 0.625$ and $b = 0.375$ (which represent the subjective estimates of the management) and the one of the balancing approach. We calculated the profit contribution for a true set of parameter values of η and b for these two alignments. Then we compared the profit contribution of COSTA's solution with the balancing approach's solution for a set of systematically varied true elasticities of potential (η) and calling time (b) in the sales response function (21). Table 5 gives the percentage profit contribution improvement of COSTA's solution over that of the balancing approach for true parameter values for the elasticities of potential (η) and calling time (b).[7]

Table 5 shows that even under severe misspecifications of the parameter values of the sales response functions, the solution of COSTA always yields profit contributions higher than those of the balancing approach. The profit improvements increase up to an elasticity of $b = 0.7$. Most probably, this stable and partly increasing superiority stems from the effect that the balancing approach assigns too many SCUs to the territories of Salespersons 1, 2, 4, and 9, which are all located in areas with rather limited sales opportunities.

5. Conclusions and Future Research

We have presented a new approach to profit contribution optimizing sales territory alignment called COSTA. It aims to revive the stream of research in the 1970s that proposed sales territory alignment decision models attempting to directly maximize profit. Such models are theoretically more appealing than the balancing approach that strives only to balance one or several attributes such as potential or workload. The balancing approach has become state-of-the-art, most probably because it is easy to understand, requires only a moderate amount of data, and feasible solutions can be produced and evaluated by hand. In contrast, the profit maximizing approaches of the 1970s were very complex, needed a large amount of data, and required specialized software that was not available at that time. To improve this situation, COSTA has been designed as a less complex model that demands less

[6]Marginal profit of selling time is determined by inserting the value for the optimized selling time into the first derivative of the profit contribution $PC_{j,r}$ in one of the SCUs in the territory of the jth salesperson.

[7]The authors greatly appreciate the suggestion of the editor and the area editor to perform this sensitivity analysis which strongly supports the superiority of COSTA.

data and solves large problems within reasonable computational time.

COSTA combines the problem of selling-time allocation with the assignment of sales coverage units (SCUs) to territories. In contrast to the profit maximization approaches of the 1970s, COSTA is based on sales response functions at the level of SCUs as opposed to individual accounts and thus requires less data. These sales response functions can be of any given concave form and incorporate travel time directly. The profit contribution objective also enables one to evaluate the profit implications of different salespersons' base locations and salesforce sizes. The applicability of COSTA has been further enhanced by the development of a very powerful solution procedure that solves even large territory alignment problems within a reasonably short time as well as offering an appropriate software.

The usefulness of COSTA has been demonstrated with the help of an application. A comparison of COSTA's results shows that COSTA produces a solution with a 3.8% higher predicted profit contribution than those of the balancing approach and which is insensitive to misspecifications of the sales response functions. Even more interesting is that, despite its justification, the balanced solution of almost equal potential across territories did not lead to equal sales and, in turn, did not lead to equal income opportunities.

Moreover, it is important to realize that territories with equal income opportunities are not a necessary prerequisite in providing fair compensation for salespersons. Rather, compensation issues can be separated from the design of sales territories by basing variable incentives not on the absolute figure of achieved sales, but on the relative figure of achieved quota. We believe that this is still overlooked by many sales managers. Another interesting result is that the marginal profits of time are not equal across territories in the profit maximizing territory alignment. Thus, equating marginal profits of time across territories will not yield a profit maximum.[8]

Appendix I: Optimal Selling Time Allocation Procedure per Territory in COSTA

Step 1: Initialization:

Given are:

j: index of salespersons for which the selling times should be allocated,

R_j: set of SCUs assigned to the jth salesperson,

$\pi_{j,opt} = 0$,

$t_{j,r}$: starting values for selling times ($r \in R_j$),

$\omega > 0$: tolerance for minimum profit improvement.

If no starting values are provided, start with a uniform allocation of selling time to SCUs of the jth salesperson, i.e.,

$$t_{j,r} = \frac{T_j}{|R_j|} \qquad (r \in R_j).$$

Step 2: Calculate profit, absolute and marginal sales, and cost as well as elasticities:

$$S_{j,r} = f_{j,r}\left(\frac{1}{1 + q_{j,r}} \cdot t_{j,r}\right) \quad \text{and} \quad C_{j,r} = h_{j,r}\left(\frac{q_{j,r}}{1 + q_{j,r}} \cdot t_{j,r}\right) \quad (r \in R_j),$$

$$\varepsilon_{j,r} = \frac{dS_{j,r}}{dt_{j,r}} \cdot \frac{t_{j,r}}{S_{j,r}} = \frac{df_{j,r}\left(\frac{1}{1 + q_{j,r}} \cdot t_{j,r}\right)}{dt_{j,r}} \cdot \frac{t_{j,r}}{S_{j,r}} \qquad (r \in R_j),$$

$$\gamma_{j,r} = \frac{dC_{j,r}}{dt_{j,r}} \cdot \frac{t_{j,r}}{C_{j,r}} = \frac{dh_{j,r}\left(\frac{q_{j,r}}{1 + q_{j,r}} \cdot t_{j,r}\right)}{dt_{j,r}} \cdot \frac{t_{j,r}}{C_{j,r}} \qquad (r \in R_j),$$

$$\mathrm{PC}_{j,r} = g_{j,r} \cdot S_{j,r} - C_{j,r} \qquad (r \in R_j),$$

$$\pi_j = \sum_{r \in R_j} \mathrm{PC}_{j,r} = \sum_{r \in R_j} (g_{j,r} \cdot S_{j,r} - C_{j,r}).$$

Step 3: Can solution be improved by better allocation of selling time to SCUs?

$$\text{If } \pi_j < \pi_{j,opt} + \omega, \text{ then go to Step 5, else } \pi_{j,opt} = \pi_j$$
$$\text{and } t_{j,r,opt} = t_{j,r} \qquad (r \in R).$$

Step 4: Reallocate selling times by inserting the current values into the optimality condition (18):

$$\mathrm{MMPC}_j = \frac{\sum_{r \in R_j} (g_{j,r} \cdot \varepsilon_{j,r} \cdot S_{j,r} - \gamma_{j,r} \cdot C_{j,r})}{T_j}$$

(for use in the assignment procedure),

$$t_{j,r} = \frac{g_{j,r} \cdot \varepsilon_{j,r} \cdot S_{j,r} - \gamma_{j,r} \cdot C_{j,r})}{\sum_{\tau \in R_j} (g_{j,\tau} \cdot \varepsilon_{j,\tau} \cdot S_{j,\tau} - \gamma_{j,\tau} \cdot C_{j,\tau})} \cdot T_j$$

[8]The authors would like to thank the editor, Professor Richard Staelin, the area editor, and three anonymous reviewers for their extremely helpful comments that have led to an improved algorithm and many other improvements of this paper. The authors would also like to thank Dietmar Kreye for implementing the algorithm and carrying out the computer trials for our simulation study as well as Knut Haase for providing the data sets for analyzing the solution quality of our algorithm with the upper bounds.

$$= \frac{g_{j,r} \cdot \varepsilon_{j,r} \cdot S_{j,r} - \gamma_{j,r} \cdot C_{j,r}}{\text{MMPC}_j} \qquad (r \in R_j).$$

Go to Step 2.

Step 5: Terminate because optimal solution is reached as indicated by a smaller improvement than the minimum tolerance ω.

Appendix II: Heuristic Solution Procedure for Assigning SCUs to Territories in COSTA

Step 1: Initialization:

$x_{j,r} = 1 \qquad (j \in J, r \in R);$
$R_j = R \quad (j \in J);$
$\pi_{\text{opt}} = 0;$
$V_r = J$ $(r \in R)$ (index set of salespersons to which the rth SCU is currently assigned);
$\text{TC}_j = 0 \quad (j \in J);$
TC_j: Total amount of reallocated selling time;
$\text{REM} = R;$
REM: Index set of SCUs assigned to more than one salesperson.

Step 2: Allocate selling times $t_{j,r}$ for all salesperson j $(j \in J)$ according to the allocation procedure.
Take $t_{j,r}$; $\text{PC}_{j,r}$ $(j \in J, r \in R)$, and MMPC_j $(j \in J)$ from the optimal solution of the allocation procedure.

Step 3: Determine that assignment (j^*,r^*) whose deletion restricts the degrees of freedom in the further search for the best feasible solution the least.

$$L_{j,r} = \text{PC}_{j,r} - t_{j,r} \cdot \text{MMPC}_j \qquad (r \in \text{REM}, j \in V_r).$$

If $|V_r| > 1$, then compute

$$LDIFF_r = \text{Max}\ \{L_{j,r} | j \in V_r\} - \text{Min}\ \{L_{j,r} | j \in V_r\} \quad (r \in \text{REM})$$

$$r^* = \text{argmax}\ \{LDIFF_r | r \in \text{REM}\}$$

$$j^* = \text{argmin}\ \{L_{j,r}{}^* | j \in V_r{}^*\}.$$

Step 4: Delete assignment (j^*,r^*).

$$x_{j^*,r^*} = 0;\ R_{j^*} = R_{j^*} \backslash r^*;\ V_{r^*} = V_{r^*} \backslash j^*;$$

$$\text{TC}_{j^*} = \text{TC}_{j^*} + t_{j^*,r^*};\ t_j{}^*{}_{,r}{}^* = 0.$$

If $|V_r{}^*| = 1$, then $\text{REM} = \text{REM} \backslash r^*$.

Step 5: Reallocate the selling time of the j^*th salesperson.
If $\text{TC}_j^* \geq \zeta \cdot T_j^*$ (where ζ is a fraction of total selling time; if the sum of selling times deleted without exact reallocation of time TC_j^* exceeds this fraction, then selling time should be reallocated exactly.)
then
$\text{TC}_j^* = 0.$
Apply the allocation procedure for the j^*th salesperson.
Take $t_{j^*,r}$, $\text{PC}_{j^*,r}$ $(r \in R_j^*)$, and MMPC_j^* from the optimal solution of the allocation procedure
else determine a heuristic reallocation

$$t_{j^*,r} := \frac{t_{j^*,r}}{(1 - t_{j^*,r^*})} \quad (r \in R_{j^*}).$$

Step 6: Is the solution feasible?
If $|\text{REM}| > 0$, then go to Step 3,

$$\text{else } \pi = \sum_{j \in J} \sum_{r \in R} PC_{j,r} \cdot x_{j,r}.$$

Step 7: Store best solution found so far.
If $\pi < \pi_{opt}$, then go to Step 8.
Apply the allocation procedure for all salespersons j $(j \in J)$ and take $t_{j,r}$, $\text{PC}_{j,r}$ $(j \in J, r \in R)$, and MMPC_j $(j \in J)$ of that solution

$$t_{j,r,\text{opt}} = t_{j,r} \text{ and } x_{j,r,\text{opt}} = x_{j,r} \qquad (j \in J, r \in R),$$
$$\pi_{\text{opt}} = \sum_{j \in J} \sum_{r \in R} PC_{j,r} \cdot x_{j,r,\text{opt}}.$$

Step 8: If solution is noncontiguous and contiguity is required, then go to Step 9, else terminate because no better solution is found.

Step 9: Initialize $x_{j,r} = x_{j,r,\text{opt}} \quad (j \in J, r \in R);$
$\pi_{\text{opt}} = 0.$
Compute R^{nc} (where R^{nc} is the set of SCUs that are not connected to the territory of their salesperson).
Set $v_r = j$ $(r \in R)$ where j is the salesperson to which the rth SCU is currently assigned.
If $|R^{nc}| = 0$, then terminate, else $x_{v_r,r} = 0 \quad (r \in R^{nc})$.
$x_{j,r} = 1$ for all SCUs $r \in R^{nc}$ and all salespersons $j(j \in J)$ that remain contiguous with these additional assignments.
Go to Step 2.

Appendix III: Derivation of Optimality Condition for the Allocation of Selling Time Across SCUs

The problem of optimally allocating selling time $t_{j,r}$ across the SCUs $(r \in R_j)$ of the jth salesperson is given by Model (15)–(17). The following Lagrange function solves problem (15)–(16):

$$L_j = \sum_{r \in R_j} \left[g_r \cdot f_{j,r} \left(\frac{1}{1 + q_{j,r}} \cdot t_{j,r} \right) - h_{j,r} \left(\frac{q_{j,r}}{1 + q_{j,r}} \cdot t_{j,r} \right) \right] - \lambda_j \cdot \left(\sum_{r \in R_j} t_{j,r} - T_j \right) \to \text{max!} \qquad \text{(A1)}$$

Partially differentiating (A1) and setting the derivatives equal to zero leads to:

$$\frac{\partial L_j}{\partial t_{j,r,\text{opt}}} = g_r \cdot \frac{\partial f_{j,r}\left(\frac{1}{1 + q_{j,r}} \cdot t_{j,r,\text{opt}}\right)}{\partial t_{j,r,\text{opt}}} - \frac{\partial h_{j,r}\left(\frac{q_{j,r}}{1 + q_{j,r}} \cdot t_{j,r,\text{opt}}\right)}{\partial t_{j,r,\text{opt}}} - \lambda_j = 0 \qquad (r \in R_j), \quad \text{(A2)}$$

$$\frac{\partial L_j}{\partial \lambda_j} = \sum_{r \in R_j} t_{j,r,\text{opt}} - T_j = 0, \qquad \text{(A3)}$$

if travel cost is small compared to profit contribution from sales such that constraint (16) is fulfilled as an equation in the optimum. Slightly rearranging and expanding Equation (A2) gives:

$$g_r \cdot \frac{\partial f_{j,r}\left(\frac{1}{1+q_{j,r}} \cdot t_{j,r,\mathrm{opt}}\right)}{\partial t_{j,r,\mathrm{opt}}} \cdot \frac{t_{j,r,\mathrm{opt}}}{S_{j,r,\mathrm{opt}}} \cdot \frac{S_{j,r,\mathrm{opt}}}{t_{j,r,\mathrm{opt}}}$$

$$- \frac{\partial h_{j,r}\left(\frac{q_{j,r}}{1+q_{j,r}}\right) \cdot t_{j,r,\mathrm{opt}}}{\partial t_{j,r,\mathrm{opt}}} \cdot \frac{t_{j,r,\mathrm{opt}}}{C_{j,r,\mathrm{opt}}} \cdot \frac{C_{j,r,\mathrm{opt}}}{t_{j,r,\mathrm{opt}}} = \lambda_j \quad (r \in R_j). \quad \text{(A4)}$$

Substituting the respective terms by the sales and cost elasticities and solving for the optimal selling time $t_{j,r,\mathrm{opt}}$ gives:

$$t_{j,r,\mathrm{opt}} = \frac{g_r \cdot \varepsilon_{j,r,\mathrm{opt}} \cdot S_{j,r,\mathrm{opt}} - \gamma_{j,r,\mathrm{opt}} \cdot C_{j,r,\mathrm{opt}}}{\lambda_j} \quad (r \in R_j). \quad \text{(A5)}$$

Expressing selling time as a fraction of total time by dividing Equation (A5) by T_j and using the information given by Equation (A3) yields:

$$\frac{t_{j,r,\mathrm{opt}}}{T_j} = \frac{t_{j,r,\mathrm{opt}}}{\sum_{r \in R_j} t_{j,r,\mathrm{opt}}}$$

$$= \frac{\dfrac{g_r \cdot \varepsilon_{j,r,\mathrm{opt}} \cdot S_{j,r,\mathrm{opt}} - \gamma_{j,r,\mathrm{opt}} \cdot C_{j,r,\mathrm{opt}}}{\lambda_j}}{\dfrac{\sum_{\tau \in R_j} (g_\tau \cdot \varepsilon_{j,\tau,\mathrm{opt}} \cdot S_{j,\tau,\mathrm{opt}} - \gamma_{j,\tau,\mathrm{opt}} \cdot C_{j,r,\mathrm{opt}})}{\lambda_j}} \quad (r \in R_j). \quad \text{(A6)}$$

Rearranging leads to:

$$t_{j,r,\mathrm{opt}} = \frac{g_r \cdot \varepsilon_{j,r,\mathrm{opt}} \cdot S_{j,r,\mathrm{opt}} - \gamma_{j,r,\mathrm{opt}} \cdot C_{j,r,\mathrm{opt}}}{\sum_{\tau \in R_j} (g_\tau \cdot \varepsilon_{j,\tau,\mathrm{opt}} \cdot S_{j,\tau,\mathrm{opt}} - \gamma_{j,\tau,\mathrm{opt}} \cdot C_{j,\tau,\mathrm{opt}})} \cdot T_j \quad (r \in R_j). \quad \text{(A7)}$$

As profit contribution from sales is assumed to be substantially higher than travel cost, the right-hand side of Equation (18) is positive such that Equation (17) is also fulfilled.

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